# Working mathematically

The Working mathematically processes in the NSW Mathematics syllabus are:

* communicating
* understanding and fluency
* reasoning
* problem solving.

Students learn to work mathematically by using these processes in an interconnected way. The coordinated development of these processes results in students becoming mathematically proficient.

When students are Working mathematically it is important to help them to reflect on how they have used their thinking to solve problems. This assists students to develop ‘mathematical habits of mind’ (Cuoco et al. 2010).

Students need many experiences that require them to relate their knowledge to the vocabulary and conceptual frameworks of mathematics.

## Overarching Working mathematically outcome

To highlight how these processes are interrelated, in Mathematics K–6 there is one overarching Working mathematically outcome.

A student develops understanding and fluency in mathematics through:

* exploring and connecting mathematical concepts
* choosing and applying mathematical techniques to solve problems
* communicating their thinking and reasoning coherently and clearly.

The Working mathematically outcome describes the thinking and doing of mathematics. In doing so, the outcome indicates the breadth of mathematical actions that teachers need to emphasise. The overarching Working mathematically outcome is the same across the K–2, 3–6 and 7–10 sections of the Mathematics syllabuses.

The Working mathematically processes should be embedded within the concepts being taught. Embedding Working mathematically ensures students are able to fluently understand concepts and make connections to other focus areas. The mathematics focus area outcomes and content provide the knowledge and skills for students to reason about, and contexts for problem solving. The overarching Working mathematically outcome is assessed in conjunction with the mathematics content outcomes. The sophistication of Working mathematically processes develops through each stage of learning and can be observed in relation to the increase in complexity of the mathematics outcomes and content. A student's level of competence in Working mathematically can be monitored over time, for example, within Additive Relations by the choice of strategy appropriate to the task, and the use of efficient strategy for the stage of learning the student is working at.

## Elaborating the processes

In becoming proficient users of mathematics, students must make connections between mathematical ideas (National Research Council 2001). For example, the connections students make between spatial and numerical representations of quantity support the development of their understanding of measurement.

Working mathematically usually involves using one or more processes. Solving problems in mathematics requires understanding of concepts and relations as well as the capacity to communicate the reasoning.

Working mathematically requires students to:

* explore and connect mathematical concepts (understanding/fluency)
* choose and apply efficient techniques to solve problems (fluency/problem solving)
* communicate their thinking and reasoning coherently and clearly (communicating/reasoning).

## Embedding the processes

The following processes are embedded within the outcomes and content:

* communicating
* understanding and fluency
* problem solving
* reasoning.

The nature of each Working mathematically process, and its place in the syllabus, is briefly described below.

### Communicating

Students need to know mathematical content and be able to express their ideas about mathematics so that others can understand them.

The ongoing interaction with mathematical vocabulary helps to reinforce students' understanding of the words and the mathematical ideas the words represent.

Students are communicating mathematically when they describe, represent, explain and reason about mathematical situations, concepts, methods and solutions.

Students communicate through a variety of ways, such as in written, oral, graphical or symbolic form, through actions, gestures or signing. Students can be supported to communicate through the use of tools such as objects or manipulatives, text with enlarged print, audio books, braille, speech-to-text and text-to-speech applications and digital technology. Making connections between representations, known as representational fluency, supports understanding of mathematics (Fonger 2019).

Oral language and communication are key skills developed in the early years of school. Guidance on developing oral language and communication is provided in the *English K–6 Syllabus*, including complementary content for alternative communication forms.

Encouraging students to reflect on and discuss the strategies they use and the knowledge and skills they require in mathematics assists them to learn to work mathematically. Students learn to think more deeply by:

* reflecting on what they have done
* organising their thoughts
* deciding how to express those thoughts.

They need opportunities to assess their own understanding, make connections and compare ideas.

### Understanding and fluency

Understanding involves making cognitive connections between new material or experience and existing ideas.

Students demonstrate conceptual understanding when they:

* connect related ideas
* represent concepts in different ways
* identify commonalities and differences between aspects of content
* describe their thinking mathematically
* use concepts to solve new and unfamiliar problems.

Conceptual understanding supports retention. Because facts and methods learnt with understanding are connected, they are easier to remember and use, and they can be reconstructed when forgotten.

Students’ communication of their understanding reveals their mathematical fluency. Mathematical fluency is developed when students (Watson and Sullivan 2008):

* choose and use appropriate strategies
* carry out procedures flexibly, accurately and efficiently
* recall factual knowledge and concepts to solve problems
* use known facts, and reason about relationships to find solutions.

### Problem solving

Some problems in mathematics can be readily solved based on experience. When confronted with a routine mathematics problem, the student is able to apply a known solution method.

In contrast, non-routine problems are problems for which the student does not immediately know a usable solution method. Questions that might start out as problematic can become routine as knowledge and experience grows. Non-routine problems require productive thinking because the student needs to invent a way to understand and solve the problem. Students need experience with both routine and non-routine mathematics problems to do this.

In becoming proficient mathematics problem-solvers, students:

* learn how to form mental representations of problems
* apply mathematical relationships
* devise novel solution methods when needed.

There is no set of generic problem-solving strategies in mathematics that students can learn to translate non-routine problems into routine problems.

Flexibility is a fundamental characteristic of problem solving in mathematics. It develops through the broadening of knowledge required for solving non-routine problems rather than just routine problems.

### Reasoning

‘Reasoning is the glue that holds everything together, the lodestar that guides learning.’ (National Research Council 2001).

Mathematics is a reasoning and creative activity. Mathematical reasoning is thinking **logically about quantitative and spatial relationships.** It is key to later achievement in mathematics (Nunes et al. 2007).

As mathematical reasoning develops, students appreciate that mathematics makes sense, and can be understood. They develop an increasingly sophisticated capacity for logical thought and actions.

Reasoning has been embedded within the outcomes and content. The syllabus also identifies useful opportunities to develop mathematical reasoning in the content. In particular, the syllabus highlights reasoning about:

* quantity
* relations
* patterns
* spatial relations
* spatial structure.

This is not an exhaustive list of when and how students may reason about mathematical ideas and concepts. These highlights assist teachers in knowing the kind of reasoning students have the opportunity to apply.

# References

Cuoco A, Goldenberg EP and Mark J (2010) ‘Contemporary Curriculum Issues: Organizing a Curriculum around Mathematical Habits of Mind’*, The Mathematics Teacher* *MT*, 103(9):682–688, [doi:10.5951/MT.103.9.0682](https://pubs.nctm.org/view/journals/mt/103/9/article-p682.xml).

Fonger NL (2019) ‘Meaningfulness in representational fluency: An analytic lens for students’ creations, interpretations, and connections’, *The Journal of Mathematical Behavior*, 54:1–26, [doi:10.1016/j.jmathb.2018.10.003](https://www.sciencedirect.com/science/article/abs/pii/S0732312318300695).

National Research Council (2001) *Adding it up: Helping children learn mathematics*, Kilpatrick J, Swafford J, and Findell B (eds), National Academies Press, Washington, DC, <doi:10.17226/9822>.

Nunes T, Bryant P, Evans D, Bell D, Gardner S, Gardner A, and Carraher JN (2007), ‘The contribution of logical reasoning to the learning of mathematics in primary school’, *British Journal of Developmental Psychology*, 25:147–166, <doi:10.1348/026151006X153127>.

Watson A and Sullivan P (2008), ‘Teachers learning about tasks and lessons’, in Tirosh D and Wood T (eds), *Tools and processes in mathematics teacher education*, Sense Publishers, Rotterdam, Netherlands.